arXiv:hep-th/9505036v2 18 May 1995

hep-th/9505036

Phase transition by curvature in three dimensional $O(N)$ sigma model

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(April, 1995)

Abstract

Using the effective potential, the large-N nonlinear $O(N)$ sigma model with the curvature coupled term is studied on $S^2 \times R^1$. We show that, for the conformally coupled case, the dynamical mass generation of the model in the strong-coupled regime on R^3 takes place for any finite scalar curvature (or radius of the S^2). If the coupling constant is larger than that of the conformally coupled case, there exist a critical curvature (radius) above (below) which the dynamical mass generation does not take place even in the strong-coupled regime. Below the critical curvature, the mass generation occurs as in the model on R^3.

04.62.+v, 11.15.Pg

Typeset using REVTEX

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I. INTRODUCTION

There has been a lot of interest in quantum field theories on curved spacetime background [1]. In general, quantum field theories may be sensitive to both the local and global structure of spacetime. The structure can alter the physical parameters in quantum loop corrections, which can not be absorbed by simple redefinition of the parameter after removing the ultraviolet divergences. A well-known example is the finite temperature field theory, where the renormalized parameters can become temperature dependent. This phenomenon could be of particular interest for the models in which the physical parameters get non-vanishing values through quantum loop corrections [2].

In this paper we will study the three dimensional nonlinear $O(N)$ sigma model [3] on $S^2 \times R^1$ Euclidean spacetime by evaluating the effective potential in the large-N limit [2,4]. In the model on R^3, there exist a critical coupling constant g_c which separates the strongly coupled case (strong-coupled regime) and weakly coupled case (weak-coupled regime), while dynamical mass generation takes place only in the strongly coupled case [3,4]. Due to S^2, the spacetime should have nonvanishing scalar curvature[1]. For the sake of explicit evaluations, we will restrict our discussions on the case of the canonical metric for S^2 and the flat metric on R^1 which gives the constant scalar curvature, $R = 2/\rho^2$, with the radius ρ of S^2. In the presence of scalar curvature, we may add a gravitational interaction term $\xi R n^i n_i$ $(= \xi R n^2)$ to Lagrangian, where ξ is a gravitational coupling constant and n^i $(i = 1, 2, \cdots, N)$ is a boson field. Taking the above arguments into consideration, the Lagrangian of the nonlinear $O(N)$ sigma model on $S^2 \times R^1$ may be written as:

$$
\begin{aligned}
L = \int d\theta d\varphi dz \rho^2 \sin\theta [\frac{1}{\rho^2}\partial_\theta n^i \partial_\theta n^i + \frac{1}{\rho^2 \sin^2\theta}\partial_\varphi n^i \partial_\varphi n_i \\
+ \partial_z n^i \partial_z n_i + \xi R n^2 + \sigma(n^2 - \frac{N}{g_0^2})],
\end{aligned}
\tag{1}
$$

where σ is a Lagrange multiplier to coerce the constraint $n^2 = N/g_0^2$ and g_0 is the bare

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[1] The author thanks Professor P. Gilkey and Dr. J.H. Park for discussions on this point.

coupling constant of the model.

In the literatures [1] special emphases have been made on the conformally coupled cases: $\xi = \frac{d-2}{4(d-1)}$ on d-dimensional spacetime or $\xi = \frac{1}{8}$ on three dimension. On S^d or $S^{d-n} \times R^n$ there have been some analyses for other models [5,6], and the ζ-function method has been mostly used in isolating divergences of quantum loop correction [1,5–8], while we will use the cut-off method.

In the next section we will evaluate the effective potential, V, on $S^2 \times R^1$ in the leading order of N. For $\xi \geq \frac{1}{8}$, the renormalized effective potential will be explicitly found. In Sec.III, we will study whether the dynamical mass generation takes place by investigating the stationary point in V. It will be shown that for $\xi = \frac{1}{8}$ dynamical mass generation will take place for any value of ρ if g is larger than g_c, as in the model on R^3. For $\xi > \frac{1}{8}$, there exists a critical curvature R_c ($= 2/\rho_c^2$); For $R \geq R_c$, dynamical mass generation does not occur even in the strongly coupled case. For $\xi > \frac{1}{8}$, we found the analytic expression of R_c, which shows that R_c decreases monotonically as ξ increases. The final section will be devoted on discussions.

II. EFFECTIVE POTENTIAL

In the leading order of the $1/N$ expansion, the effective potential for constant σ is given by the tree and one-loop diagrams with external σ lines [9,2]. Or, in order to make use of functional integral [10], one can write the Lagrangain density with the quantum mechanical angular momentum operator $\mathbf{L}$ as follows;

$$\mathcal{L} = n^i D n_i - N\sigma/g_0^2, \tag{2}$$

where

$$D = -\partial_z^2 + \frac{\mathbf{L}^2}{\rho^2} + \frac{2\xi}{\rho^2} + \sigma. \tag{3}$$

By performing the Gaussian functional integral, one can find the effective potential per unit volume as

$$\frac{V_0}{N} = -\frac{\sigma}{g_0^2} + \frac{1}{4\pi\rho^2}(\mathrm{Tr}\ln D + C). \tag{4}$$

C is a constant which may arise from the functional integral and it will be fixed by demanding that $V_0 \mid_{\sigma=0}= 0$. As is well-known, the diagrams which have σ propagators as internal lines give contributions of next to the leading order in the $1/N$ expansion. Therefore, the effective potential of the leading order per unit volume can be written as;

$$\frac{V}{N} = -\frac{\sigma}{g_0} + \frac{1}{4\pi\rho^2}\sum_{l=0}^{I}\int_{|k|<\Lambda_1}\frac{dk}{2\pi}(2l+1)\ln(1+\frac{\sigma}{k^2+\frac{l(l+1)}{\rho^2}+\frac{2\xi}{\rho^2}+i\epsilon}). \tag{5}$$

In Eq.(5), we introduce the cut-off Λ_1 and I. While the mass dimension of Λ_1 is that of a momentum as usual, the I is a pure number since it is a cut-off for the quantum number of the operator $\mathbf{L}$. If sapcetime is a product of two manifolds with different topologies, it is necessary to introduce different cut-offs. Although this should happen in finite-temperature field theories, this could be implicit if summation or integral along some directions are finite.

In order to compare the effective potential in Eq.(5) with that on R^3, one can use the following formula:

$$\sum_{l=0}^{N} f(l) = \frac{1}{2}f(0) + \int_0^{N+1} f(x)dx + \sum_{l=0}^{N}\int_0^1 f'(x+l)(x-\frac{1}{2})dx - \frac{1}{2}f(N+1), \tag{6}$$

which comes from the Euler-Maclaurin formula

$$\int_0^1 f(x+l)dx = \frac{1}{2}(f(l+1)+f(l)) - \int_0^1 f'(x+l)(x-\frac{1}{2})dx \tag{7}$$

for any differentiable function $f(x)$. Making use of the equality in Eq.(6), the effective potential can be written as

$$\begin{aligned}\frac{V}{N} = &-\frac{\sigma}{g_0^2} + \frac{1}{8\pi^2\rho^2}\int_{|k|<\Lambda_1}\int_{x=0}^{I} dx(2x+1)\ln(1+\frac{\rho^2\sigma}{(x+\frac{1}{2})^2+2(\xi-\frac{1}{8})+\rho^2k^2+i\epsilon}) \\ &+\frac{1}{16\pi^2\rho^2}\int_{-\infty}^{\infty} dk\ln(1+\frac{\rho^2\sigma}{\rho^2k^2+2\xi+i\epsilon}) \\ &+\frac{1}{8\pi^2}\int_{-\infty}^{\infty} dk\sum_{l=0}^{\infty}\int_0^1 dx(x-\frac{1}{2})\left[\begin{array}{l}\frac{2}{R^2}\ln(1+\frac{\rho^2\sigma}{(x+\frac{1}{2}+l)^2+2(\xi-\frac{1}{8})+\rho^2k^2+i\epsilon}) \\ -\frac{4\sigma(x+\frac{1}{2}+l)^2}{[(x+\frac{1}{2}+l)^2+2(\xi-\frac{1}{8})+\rho^2k^2+i\epsilon][(x+\frac{1}{2}+l)^2+2(\xi-\frac{1}{8})+\rho^2k^2+\rho^2\sigma+i\epsilon]}\end{array}\right] \\ &+O(1/\Lambda_1)+O(1/I)\end{aligned} \tag{8}$$

It is convenient to divide V into two pieces so that

$$V = V_\xi + V_\Delta + O(1/\Lambda_1) + O(1/I), \tag{9}$$

where

$$\frac{V_\xi}{N} = -\frac{\sigma}{g_0^2} + \frac{1}{8\pi^3}\int_{|k|<\Lambda_1} dk \int_{k_2=0}^{\Lambda_2} dk_2\ 2\pi k_2 \ln(1 + \frac{\sigma}{k^2 + k_2^2 + \frac{2(\xi-\frac{1}{8})}{\rho^2} + i\epsilon}) \tag{10}$$

and

$$\begin{aligned}\frac{V_\Delta}{N} &= \frac{1}{16\pi^2\rho^2}\int_{-\infty}^{\infty} dk \ln(1 + \frac{\sigma}{k^2 + \frac{2\xi}{\rho^2} + i\epsilon}) \\ &\quad -\frac{1}{4\pi^2\rho^2}\int_0^{\frac{1}{2}} dt\ t \int_{-\infty}^{\infty} dk \ln(1 + \frac{\sigma}{k^2 + (\frac{t}{\rho})^2 + \frac{2}{\rho^2}(\xi - \frac{1}{8}) + i\epsilon}) \\ &\quad +\frac{1}{8\pi^2}\sum_{l=0}^{\infty}\int_{-\frac{1}{2}}^{\frac{1}{2}} dt\ t \int_{-\infty}^{\infty} dk \left[\begin{array}{l} \frac{2}{R^2}\ln(1 + \frac{\sigma}{k^2 + \frac{(t+l)^2}{\rho^2} + \frac{2(\xi-\frac{1}{8})}{\rho^2} + i\epsilon}) \\ -\frac{4\sigma(t+l)^2/\rho^4}{[k^2 + \frac{(t+l)^2}{\rho^2} + \frac{2(\xi-\frac{1}{8})}{\rho^2} + i\epsilon][k^2 + \frac{(t+l)^2}{\rho^2} + \sigma + \frac{2(\xi-\frac{1}{8})}{\rho^2} + i\epsilon]}\end{array}\right].\end{aligned} \tag{11}$$

In Eq.(10), the Λ_2 whose dimension is of momentum is defined as

$$\Lambda_2 = \frac{I + \frac{1}{2}}{\rho}.$$

From now on we will restrict our attention on the case $\xi \geq \frac{1}{8}$, for which logarithm functions or their integrals in Eqs.(8,10,11) are well defined even when $\epsilon = 0$. For discussions on the case of $\xi < \frac{1}{8}$, we should develop some analytic continuations for logarithm functions, which is beyond the scope of this paper.

Making use of the formulae [11]

$$\int_{-\infty}^{\infty} \ln\frac{\alpha^2 + x^2}{\beta^2 + x^2} dx = 2(|\alpha| - |\beta|)\pi, \tag{12}$$

$$\int_{-\infty}^{\infty} \frac{dx}{(\gamma + x^2)(\delta + x^2)} = \frac{\pi}{\sqrt{\gamma\delta}(\sqrt{\gamma} + \sqrt{\delta})} = \frac{\pi}{\sqrt{\gamma\delta}}\frac{\sqrt{\delta} - \sqrt{\gamma}}{\delta - \gamma}, \tag{13}$$

we can find a simpler form of V_Δ;

$$
\begin{aligned}
V_\Delta = & -\frac{1}{2\pi\rho^2}\int_0^{\frac{1}{2}} dt\ t[\sqrt{(\frac{t}{\rho})^2+\frac{2(\xi-\frac{1}{8})}{\rho^2}+\sigma}-\sqrt{(\frac{t}{\rho})^2+\frac{2(\xi-\frac{1}{8})}{\rho^2}}] \\
& +\frac{1}{8\pi\rho^2}[\sqrt{\frac{2\xi}{\rho^2}+\sigma}-\frac{\sqrt{2\xi}}{\rho}] \\
& +\frac{1}{2\pi}\sum_{l=1}^{\infty}\int_{-\frac{1}{2}}^{\frac{1}{2}} dt\ t\left[\begin{array}{l} \frac{1}{\rho^2}[\sqrt{\frac{(t+l)^2}{\rho^2}+\frac{2(\xi-\frac{1}{8})}{\rho^2}+\sigma}-\sqrt{\frac{(t+l)^2}{\rho^2}+\frac{2(\xi-\frac{1}{8})}{\rho^2}}] \\ -\frac{\sigma(t+l)^2}{\rho\sqrt{[(t+l)^2+2(\xi-\frac{1}{8})+\rho^2\sigma][(t+l)^2+2(\xi-\frac{1}{8})]}} \\ \times\frac{1}{\sqrt{[(t+l)^2+2(\xi-\frac{1}{8})+\rho^2\sigma}+\sqrt{[(t+l)^2+2(\xi-\frac{1}{8})}} \end{array}\right] \qquad (14) \\
= & -\frac{1}{2\pi\rho^3}\int_0^{\frac{1}{2}} dt\ t[\sqrt{t^2+2(\xi-\frac{1}{8})+\rho^2\sigma}-\sqrt{t^2+2(\xi-\frac{1}{8})}] \\
& +\frac{1}{8\pi\rho^3}[\sqrt{2\xi+\rho^2\sigma}-\sqrt{2\xi}] \\
& +\frac{1}{2\pi\rho^3}\sum_{l=1}^{\infty}\int_{-\frac{1}{2}}^{\frac{1}{2}} dt\ t\left[\begin{array}{l} [\sqrt{(t+l)^2+2(\xi-\frac{1}{8})+\rho^2\sigma}-\sqrt{(t+l)^2+2(\xi-\frac{1}{8})}] \\ \times[1-\frac{(t+l)^2}{\sqrt{(t+l)^2+2(\xi-\frac{1}{8})+\rho^2\sigma}\sqrt{(t+l)^2+2(\xi-\frac{1}{8})}}] \end{array}\right], \qquad (15)
\end{aligned}
$$

which clearly shows that V_Δ is finite for any ρ, σ. Furthermore V_Δ is zero in the limit ρ approaches to infinity (in the R^3 limit), which shows that it is an effect of topology. In this limit, the integral of Eq.(10) for V_ξ which diverges with infinite cut-offs, in fact, corresponds to that of the model on R^3:

$$
\frac{1}{8\pi^3}\int dk \int dk_2{}_{k_2>0, k^2+k_2^2<\Lambda^2}\ 2\pi k_2 \ln(1+\frac{\sigma}{k^2+k_2^2+i\epsilon}).
$$

The difference in the R^3-limit is that we have two cut-offs, because our spacetime $S^2\times R^1$ is the product of two spaces with different topologies. As mentioned earlier this should happen, for example, in finite-temperature field theory, if we use cut-off regularization. In renormalizing the model on $S^2\times R^1$ we will assume - as usual - that we could deform the momentum space of integration. That is, the V_ξ which does not vanish in the R^3-limit could be written as:

$$
\begin{aligned}
\frac{V_\xi}{N} = & -\frac{\sigma}{g_0^2}+\frac{1}{8\pi^3}\int dk \int dk_2{}_{k_2>0, k^2+k_2^2<\Lambda^2}\ 2\pi k_2 \ln(1+\frac{\sigma}{k^2+k_2^2+\frac{2(\xi-\frac{1}{8})}{\rho^2}+i\epsilon}) \qquad (16) \\
= & -\frac{\sigma}{g_0^2}+\frac{\sigma\Lambda}{2\pi^2}-\frac{1}{6\pi}\sqrt{[\frac{2(\xi-\frac{1}{8})}{\rho^2}+\sigma]^3}+\frac{1}{3\pi\rho^3}\sqrt{2(\xi-\frac{1}{8})^3} \\
& +O(1/\Lambda), \qquad (17)
\end{aligned}
$$

which shows that the divergence of the potential on $S^2 \times R^1$ (or Λ dependence) is the same as that on R^3 and we could renormalize the model on $S^2 \times R^1$ by using the relation on R^3 [4]:

$$\frac{1}{g_0^2} = \frac{1}{g^2} + \int_{|p|<\Lambda} \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + M^2} \tag{18}$$

$$= \frac{1}{g^2} + \frac{\Lambda}{2\pi} - \frac{M}{4\pi} + O(1/\Lambda), \tag{19}$$

where M is the renormalization mass.

The fact that topological change of spacetime does not give rise to new counterterms in the renormalization has been found explicitly for some cases (for example, see Ref. [5]) and a more general discussion has been given in Ref. [12].

Now we have the renormalized effective potential for $\xi \geq \frac{1}{8}$

$$\frac{V}{N} = -\frac{\sigma}{g^2} + \frac{\sigma M}{4\pi} - \frac{1}{6\pi}[\sqrt{[\frac{2(\xi - \frac{1}{8})}{\rho^2} + \sigma]^3} + \frac{1}{3\pi\rho^3}\sqrt{2(\xi - \frac{1}{8})^3}] + \frac{V_\Delta}{N} + O(1/\Lambda), \tag{20}$$

which, in the R^3-limit, reduces to the potential on R^3, V_0 [4]:

$$\frac{V_0}{N} = -\frac{\sigma}{g^2} + \frac{\sigma M}{4\pi} - \frac{\sigma^{3/2}}{6\pi} + O(1/\Lambda). \tag{21}$$

As a preliminary of the next section, it may be good to recapitulate the dynamical properties of the model on R^3. The first derivative of V_0 with respect to σ

$$\frac{1}{N}\frac{\partial V_0}{\partial \sigma} = \frac{M}{4\pi} - \frac{1}{g^2} - \frac{\sqrt{\sigma}}{4\pi} + O(1/\Lambda)$$

shows that there exist a global stationary point only when $g^2 > \frac{4\pi}{M}(= g_c^2)$. That is, dynamical mass generation takes place only in the strongly coupled case ($g^2 > g_c^2$), and g_c is the critical coupling constant. The dynamically generated mass is $\sqrt{\sigma_0}$ ($= M - \frac{4\pi}{g^2}$) which reduces to zero as g goes to g_c [13].

III. PHASE STRUCTURE

Though the forms of V_0 and V differ by V_Δ which is rather complicated, they share the facts that $V_0 \mid_{\sigma=0} = V \mid_{\sigma=0} = 0$ and

$$V \simeq V_0 \simeq -\frac{\sigma^{3/2}}{6\pi} \quad \text{for large } \sigma;$$

That is, they start from 0 and decrease to $-\infty$ as σ increases to ∞. To see the shape of V, we evaluate the first derivative of V with respect to σ;

$$\frac{\rho}{N}\frac{\partial V}{\partial \sigma} = (\frac{M}{4\pi} - \frac{1}{g^2})\rho - f(\xi; y), \tag{22}$$

where

$$\begin{aligned} f(\xi; y) &= \frac{1}{4\pi}\sqrt{2(\xi - \frac{1}{8}) + y} - \frac{1}{16\pi}\frac{1}{\sqrt{2\xi + y}} + \frac{1}{4\pi}\int_0^{\frac{1}{2}} dt\ t \frac{1}{\sqrt{t^2 + 2(\xi - \frac{1}{8}) + y}} \\ &\quad - \frac{1}{4\pi}\sum_{l=1}^{\infty}\int_{-\frac{1}{2}}^{\frac{1}{2}} dt\ t \begin{bmatrix} \frac{1}{\sqrt{(t+l)^2+2(\xi-\frac{1}{8})+y}}[1 - \frac{(t+l)^2}{\sqrt{(t+l)^2+2(\xi-\frac{1}{8})+y}\sqrt{(t+l)^2+2(\xi-\frac{1}{8})}}] \\ + \frac{(t+l)^2}{\sqrt{[(t+l)^2+2(\xi-\frac{1}{8})+y]^3[(t+l)^2+2(\xi-\frac{1}{8})]}} \\ \times[\sqrt{(t+l)^2 + 2(\xi - \frac{1}{8}) + y} - \sqrt{(t+l)^2 + 2(\xi - \frac{1}{8})}] \end{bmatrix} \\ &= \frac{1}{4\pi}\sqrt{2(\xi - \frac{1}{8}) + y} - \frac{1}{16\pi}\frac{1}{\sqrt{2\xi + y}} + \frac{1}{4\pi}\int_0^{\frac{1}{2}} dt\ t \frac{1}{\sqrt{t^2 + 2(\xi - \frac{1}{8}) + y}} \\ &\quad - \frac{2(\xi - \frac{1}{8}) + y}{4\pi}\sum_{l=1}^{\infty}\int_{-\frac{1}{2}}^{\frac{1}{2}} dt\ t \frac{t}{[(t+l)^2 + 2(\xi - \frac{1}{8}) + y]^{3/2}} \end{aligned} \tag{23}$$

with $y = \rho^2\sigma$. Making use of the formulae in the appendix, a further simplification of f is possible;

$$f(\xi; y) = \frac{1}{4\pi}\mathcal{F}_{\frac{1}{2}}(\sqrt{2(\xi - \frac{1}{8}) + y}) = \frac{1}{4\pi}[\mathcal{F}(2\sqrt{2(\xi - \frac{1}{8}) + y}) - \mathcal{F}(\sqrt{2(\xi - \frac{1}{8}) + y})], \tag{24}$$

where for $\alpha > 0$ $\mathcal{F}_{\frac{1}{2}}(\alpha)$ and $\mathcal{F}(\alpha)$ are defined by

$$\begin{aligned} \mathcal{F}_{\frac{1}{2}}(\alpha) &= \sum_{l=0}^{\infty}[1 - \frac{l + \frac{1}{2}}{\sqrt{\alpha^2 + (l + \frac{1}{2})^2}}], \\ \mathcal{F}(\alpha) &= \sum_{l=1}^{\infty}[1 - \frac{1}{\sqrt{1 + (\frac{\alpha}{l})^2}}]. \end{aligned} \tag{25}$$

It is easy to find that $\mathcal{F}(0) = 0$ and $\mathcal{F}_{\frac{1}{2}}(\alpha)$ monotonically increases as α increases, which proves that if there is a stationary point of V then it must be global. Since the dynamical mass generation is denoted by the presence of stationary point of the effective potential, the above facts reveal two properties of the model on $S^2 \times R^1$ for $\xi \geq \frac{1}{8}$. The first is that if

dynamical mass generation takes place it occurs only in the strong-coupled regime ($g^2 > g_c^2$), since $f(\xi; y)$ is never less than 0. Secondly, even in the strong-coupled regime dynamical mass generation takes place only if ρ (R) of 2-sphere is larger (smaller) than ρ_c (R_c). Since $f(\xi; y)$ is monotonically increasing function of y for a fixed ξ, the critical condition is given by

$$\frac{\partial V}{\partial \sigma} \mid_{\sigma=0} = 0, \tag{26}$$

which gives the ρ_c as;

$$\rho_c \sqrt{\sigma_0} = \mathcal{F}_{\frac{1}{2}}(\sqrt{2\xi - \frac{1}{4}}) = \mathcal{F}(2\sqrt{2\xi - \frac{1}{4}}) - \mathcal{F}(\sqrt{2\xi - \frac{1}{4}}). \tag{27}$$

The smallest ρ_c (largest R_c) which is in the the conformal coupling case ($\xi = \frac{1}{8}$), is 0 (∞) since $\mathcal{F}(0)$ is zero. In other words, the dynamical mass generation takes place for any size of sphere in the strong-coupled regime of the conformal coupling case.

IV. CONCLUSION

We have studied the large-N nonlinear $O(N)$ sigma model on $S^2 \times R^1$ by evaluating the effective potential with cut-off method. By analysing the (renormalized) effective potential for $\xi \geq \frac{1}{8}$, we find that in the strongly coupled case there exists critical size or curvature of 2-sphere. Even in the strongly coupled case the dynamical mass generation does not take place when the radius (curvature) of two sphere is smaller (larger) than the critical one.

As the change of temperature may cause phase transition in finite-temperature field theory, it has been known that the change of curvature of background spacetime could give rise to phase transition [14,8], while our results provide another explicit example. The smallest critical radius (largest critical curvature) for the model we considered is 0 (∞) in conformal (gravitational) coupling case. Thus on $S^2 \times R^1$ dynamical mass generation of the model on R^3 always takes place in the conformal coupling case.

The value of σ at which the potential is stationary is the square of the dynamically generated mass. Though we cannot analytically find the value of dynamically generated

mass, the fact that $f(\xi; y)$ is a monotonically increasing function of y suggests that the phase transition would be of second order, since the mass continuously approach zero as ρ goes to ρ_c from the above ($\rho > \rho_c$). For the conformal coupling case ($\xi = 1/8$), $f(\frac{1}{8}; y)$ is also a monotonically increasing function of y. Therefore, at $\xi = 1/8$ the dynamically generated mass approaches 0 as g goes to g_c, which is support the ansatz made in Ref. [13].

It would be interesting to analyse the case $\xi < \frac{1}{8}$ which is beyond the scope of this paper. It would be also of great interest to analyse the model on different topologies. A similar analysis of the model on S^2 will be published elsewhere [15].

ACKNOWLEDGMENTS

The author thanks Professor L.H. Ford for fruitful discussion on his works and Professor G. 't Hooft for discussion on treatment of cut-offs. He also thanks Dr. W. Bietenholz for kind comments on the first version of this paper. This work was supported in part by the KOSEF and also by the U.S. Department of Energy under Contract No. DE-AC02-76-ER03130.

APPENDIX

In Sec. III the following facts are used;

$$\sum_{l=1}^{\infty} \int_{-\frac{1}{2}}^{\frac{1}{2}} dt \frac{t}{[(t+l)^2+\alpha^2]^{3/2}} = I_1 + I_2, \tag{28}$$

where

$$I_1 = \sum_{l=1}^{\infty} \int_{l-\frac{1}{2}}^{l+\frac{1}{2}} \frac{y}{[y^2+\alpha^2]^{3/2}} dy = \frac{1}{\sqrt{\frac{1}{4}+\alpha^2}} \tag{29}$$

and

$$\begin{aligned}
\alpha^2 I_2 &= \sum_{l=1}^{\infty} l [\frac{l-\frac{1}{2}}{\sqrt{\alpha^2+(l-\frac{1}{2})^2}} - \frac{l+\frac{1}{2}}{\sqrt{\alpha^2+(l+\frac{1}{2})^2}}] \\
&= \frac{1}{2\sqrt{\alpha^2+\frac{1}{4}}} + \lim_{N\to\infty} [\sum_{l=1}^{N} \frac{l+\frac{1}{2}}{\sqrt{\alpha^2+(l+\frac{1}{2})^2}} - (N+1)\frac{N+\frac{3}{2}}{\sqrt{\alpha^2+(N+\frac{3}{2})^2}}] \\
&= \sum_{l=0}^{\infty} [\frac{l+\frac{1}{2}}{\sqrt{\alpha^2+(l+\frac{1}{2})^2}} - 1] = -\mathcal{F}_{\frac{1}{2}}(\alpha).
\end{aligned} \tag{30}$$

$\mathcal{F}_{\frac{1}{2}}(\alpha)$ and $\mathcal{F}(\alpha)$ are related as follows;

$$\mathcal{F}_{\frac{1}{2}}(\alpha) = \mathcal{F}(2\alpha) - \mathcal{F}(\alpha). \tag{31}$$

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